4.4 The MVT for integrals (average value over an interval)

and the

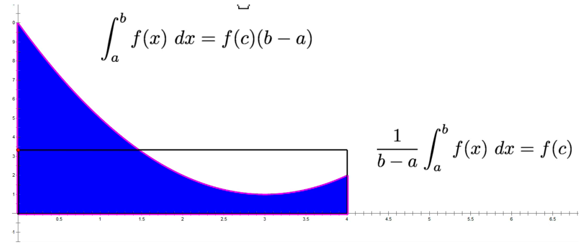
1st & 2nd FTC

1

Average Value on a Interval

Mean Value Theorem for Integrals

$$\int_a^b f(x)\ dx = f(c)(b-a)$$

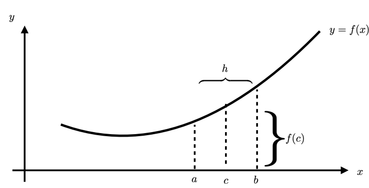


$$\frac{1}{b-a}\int_a^b f(x)\ dx = f(c)$$

2

There must be a c in $[a,b]$ where

$$f(c)\cdot(b-a) = \int_a^b f(x)\ dx$$



$f(c)$ is the average value of f on the interval $[a,b]$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\ dx$$

3

4.4 The Fundamental Theorem of Calculus

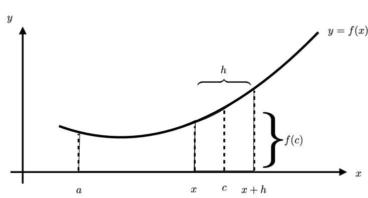
$$\int_a^b f(x)\ dx = F(b) - F(a)$$

4

Claim:

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

The proof...



$A(x) = \int_a^x f(t)\ dt$ **... and ...** $\int f(x)\ dx = F(x) + C$, if $F'(x) = f(x)$

$A'(x) = \lim_{h\to 0}\frac{A(x+h)-A(x)}{h} = \lim_{h\to 0}\frac{f(c)\cdot\cancel{h}}{\cancel{h}} = \lim_{h\to 0} f(c) = f(x)$ so $A(x)$ is an antiderivative of $f(x)$

$A(x) = F(x) + C = \int_a^x f(t)\ dt$ consider $A(a) = \int_a^a f(t)\ dt = 0 = F(x) + C$

Here $x = a$ so $F(a) + C = 0$

Then $\int_a^x f(t)\ dt = F(x) - F(a)$ ... so $C = -F(a)$

If we let $x = b$ we have the FTC: $\int_a^b f(x)\ dx = F(b) - F(a)$

5

The Second Fundamental Theorem of Calculus

$$\frac{d}{dx}\left[\int f(x)\ dx\right] = f(x)$$

6

The Net Change "Theorem"

$$\int_a^b F'(x)\ dx = F(b) - F(a)$$

or

$$F(a) + \int_a^b F'(x)\ dx = F(b)$$

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